

Gravitational waves from the R^{-1} high order theory of gravity

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Abstract

This paper is a review of a previous research on gravitational waves from the R^{-1} high order theory of gravity. It is shown that a massive scalar mode of gravitational waves from the R^{-1} theory generates a longitudinal force in addition of the transverse one which is proper of the massless gravitational waves and the response of an arm of an interferometer to this longitudinal effect in the frame of a local observer is computed. Important consequences from a theoretical point of view could arise from this approach, because it opens to the possibility of using the signals seen from interferometers to understand which is the correct theory of gravitation.

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1 Introduction

The data analysis of interferometric gravitational waves (GWs) detectors has recently started (for the current status of GWs interferometers see [1, 2, 3, 4, 5,

6, 7, 8]) and the scientific community hopes in a first direct detection of GWs in next years.

Detectors for GWs will be important for a better knowledge of the Universe and also to confirm or ruling out the physical consistency of General Relativity or of any other theory of gravitation [9, 10, 11, 12, 13, 14]. This is because, in the context of Extended Theories of Gravity, some differences between General Relativity and the others theories can be pointed out starting by the linearized theory of gravity [15, 16, 17, 18, 19, 20].

In this paper, which is a review of a previous research on gravitational waves from the R^{-1} high order theory of gravity [15], it is shown that massive scalar modes of gravitational waves from the R^{-1} theory generate a longitudinal force in addition of the transverse one which is proper of the massless gravitational waves and the response of an arm of an interferometer to this longitudinal effect in the frame of a local observer is computed. Important consequences from a theoretical point of view could arise from this approach, because it opens to the possibility of using the signals seen from interferometers to understand which is the correct theory of gravitation.

2 A scalar massive mode of gravitational radiation in the R^{-1} high order theory of gravity

In [15] it has been shown that a massive scalar mode of gravitational radiation arises from the high order action

$$S = \int d^4x \sqrt{-g} R^{-1} + \mathcal{L}_m, \quad (1)$$

where R is the Ricci scalar curvature. Equation (1) is a particular choice with respect the well known canonical one of General Relativity (the Einstein - Hilbert action [21, 22]) which is

$$S = \int d^4x \sqrt{-g} R + \mathcal{L}_m. \quad (2)$$

From the linearized field equations arising by the action (1), in [15] it has been obtained a plane wave propagating in the z direction:

$$h_{\mu\nu}(t, z) = A^+(t - z)e_{\mu\nu}^{(+)} + A^\times(t - z)e_{\mu\nu}^{(\times)} + \Phi(t - v_G z)\eta_{\mu\nu}. \quad (3)$$

The term $A^+(t - z)e_{\mu\nu}^{(+)} + A^\times(t - z)e_{\mu\nu}^{(\times)}$ describes the two standard (i.e. tensorial) polarizations of gravitational waves which arise from General Relativity, while the term $\Phi(t - v_G z)\eta_{\mu\nu}$ is the scalar massive field arising from the high order theory.

3 A longitudinal force

For a purely scalar gravitational wave eq. (3) can be rewritten as [15]

$$h_{\mu\nu}(t - v_G z) = \Phi(t - v_G z)\eta_{\mu\nu} \quad (4)$$

and the correspondent line element is the conformally flat one

$$ds^2 = [1 + \Phi(t - v_G z)](-dt^2 + dz^2 + dx^2 + dy^2). \quad (5)$$

But, in a laboratory environment on Earth, the coordinate system in which the space-time is locally flat [12, 15, 21, 22] is typically used and the distance between any two points is given simply by the difference in their coordinates in the sense of Newtonian physics. This frame is the proper reference frame of a local observer, located for example in the position of the beam splitter of an interferometer. In this frame gravitational waves manifest themselves by exerting tidal forces on the masses (the mirror and the beam-splitter in the case of an interferometer). The effect of the gravitational wave on test masses is described in this frame by the equation

$$\ddot{x}^i = -\tilde{R}_{0k0}^i x^k, \quad (6)$$

which is the equation for geodesic deviation.

But, because the linearized Riemann tensor $\tilde{R}_{\mu\nu\rho\sigma}$ is invariant under gauge transformations [12, 15, 21, 22], it can be directly computed from eq. (4).

From [21] it is:

$$\tilde{R}_{\mu\nu\rho\sigma} = \frac{1}{2}\{\partial_\mu\partial_\beta h_{\alpha\nu} + \partial_\nu\partial_\alpha h_{\mu\beta} - \partial_\alpha\partial_\beta h_{\mu\nu} - \partial_\mu\partial_\nu h_{\alpha\beta}\}, \quad (7)$$

that, in the case eq. (4), begins [15]

$$\tilde{R}_{0\gamma 0}^\alpha = \frac{1}{2}\{\partial^\alpha\partial_0\Phi\eta_{0\gamma} + \partial_0\partial_\gamma\Phi\delta_0^\alpha - \partial^\alpha\partial_\gamma\Phi\eta_{00} - \partial_0\partial_0\Phi\delta_\gamma^\alpha\}. \quad (8)$$

The computation has been performed in details in [15], the results are

$$\begin{aligned} \tilde{R}_{010}^1 &= -\frac{1}{2}\ddot{\Phi} \\ \tilde{R}_{010}^2 &= -\frac{1}{2}\ddot{\Phi} \\ \tilde{R}_{030}^3 &= \frac{1}{2}m^2\Phi, \end{aligned} \quad (9)$$

which show that the field is not transversal.

Infact, using eq. (6) it results

$$\ddot{x} = \frac{1}{2}\ddot{\Phi}x, \quad (10)$$

$$\ddot{y} = \frac{1}{2}\ddot{\Phi}y \quad (11)$$

and

$$\ddot{z} = -\frac{1}{2}m^2\Phi(t - v_G z)z. \quad (12)$$

Then the effect of the mass is the generation of a *longitudinal* force (in addition to the transverse one). Note that in the limit $m \rightarrow 0$ the longitudinal force vanishes.

4 The interferometer's response to the longitudinal component

We have to recall that the scalar wave needs a frequency which falls in the frequency-range for earth based gravitational antennas [12, 15], that is the interval $10Hz \leq f \leq 10KHz$ (see refs [1, 2, 3, 4, 5, 6, 7, 8]). For a massive scalar gravitational wave, this implies [12, 15]

$$0eV \leq m \leq 10^{-11}eV. \quad (13)$$

Equations (10), (11) and (12) give the tidal acceleration of the test mass caused by the scalar gravitational wave respectively in the x direction, in the y direction and in the z direction [15].

Equivalently we can say that there is a gravitational potential [12, 15]:

$$V(\vec{r}, t) = -\frac{1}{4}\ddot{\Phi}(t - \frac{z}{v_P})[x^2 + y^2] + \frac{1}{2}m^2 \int_0^z \Phi(t - v_G z) da, \quad (14)$$

which generates the tidal forces, and that the motion of the test mass is governed by the Newtonian equation

$$\ddot{\vec{r}} = -\nabla V. \quad (15)$$

To obtain the longitudinal component of the scalar gravitational wave the solution of eq. (12) has to be found.

For this goal the perturbation method can be used. A function of time for a fixed z , $\psi(t - v_G z)$, can be defined [12, 15], for which it is

$$\ddot{\psi}(t - v_G z) \equiv \Phi(t - v_G z) \quad (16)$$

(note: the most general definition is $\psi(t - v_G z) + a(t - v_G z) + b$, but, assuming only small variations in the positions of the test masses, it results $a = b = 0$).

In this way it results

$$\delta z(t - v_G z) = -\frac{1}{2}m^2 z_0 \psi((t - v_G z)). \quad (17)$$

A feature of the frame of a local observer is the coordinate dependence of the tidal forces due by scalar gravitational waves which can be changed with a mere shift of the origin of the coordinate system [12, 15]:

$$x \rightarrow x + x', \quad y \rightarrow y + y' \text{ and } z \rightarrow z + z'. \quad (18)$$

The same applies to the test mass displacements in the z direction, eq. (17). This is an indication that the coordinates of a local observer are not simple as they could seem [12, 15].

Now, let us consider the relative motion of test masses. A good way to analyze variations in the proper distance (time) of test masses is by means of “bouncing photons” [12, 15]. A photon can be launched from the beam-splitter to be bounced back by the mirror. It will be assumed that both the beam-splitter and the mirror are located along the z axis of our coordinate system (i.e. an arm of the interferometer is in the z direction, which is the direction of the propagating massive scalar gravitational wave and of the longitudinal force).

In the frame of a local observer, two different effects have to be considered in the calculation of the variation of the round-trip time for photons [12, 15]. The unperturbed coordinates for the beam-splitter and the mirror are $z_b = 0$ and $z_m = L$. So the unperturbed propagation time between the two masses is

$$T = L. \quad (19)$$

From eq. (17) it results that the displacements of the two masses under the influence of the scalar gravitational wave are

$$\delta z_b(t) = 0 \quad (20)$$

and

$$\delta z_m(t - v_GL) = -\frac{1}{2}m^2 L \psi(t - v_GL). \quad (21)$$

In this way, the relative displacement, is

$$\delta L(t) = \delta z_m(t - v_GL) - \delta z_b(t) = -\frac{1}{2}m^2 L \psi(t - v_GL), \quad (22)$$

Thus it results

$$\frac{\delta L(t)}{L} = \frac{\delta T(t)}{T} = -\frac{1}{2}m^2 \psi(t - v_GL). \quad (23)$$

But there is the problem that, for a large separation between the test masses (in the case of Virgo or LIGO the distance between the beam-splitter and the mirror is three or four kilometers), the definition (22) for relative displacement becomes unphysical because the two test masses are taken at the same time and therefore cannot be in a casual connection [12, 15]. The correct definitions for our bouncing photon can be written like

$$\delta L_1(t) = \delta z_m(t - v_GL) - \delta z_b(t - T_1) \quad (24)$$

and

$$\delta L_2(t) = \delta z_m(t - v_G L - T_2) - \delta z_b(t), \quad (25)$$

where T_1 and T_2 are the photon propagation times for the forward and return trip correspondingly. According to the new definitions, the displacement of one test mass is compared with the displacement of the other at a later time to allow for finite delay from the light propagation. Note that the propagation times T_1 and T_2 in eqs. (24) and (25) can be replaced with the nominal value T because the test mass displacements are already first order in Φ . Thus, for the total change in the distance between the beam splitter and the mirror in one round-trip of the photon, it is

$$\delta L_{r.t.}(t) = \delta L_1(t - T) + \delta L_2(t) = 2\delta z_m(t - v_G L - T) - \delta z_b(t) - \delta z_b(t - 2T), \quad (26)$$

and in terms of the amplitude and mass of the SGW:

$$\delta L_{r.t.}(t) = -m^2 L \psi(t - v_G L - T). \quad (27)$$

The change in distance (27) leads to changes in the round-trip time for photons propagating between the beam-splitter and the mirror:

$$\frac{\delta_1 T(t)}{T} = -m^2 \psi(t - v_G L - T). \quad (28)$$

In the last calculation (variations in the photon round-trip time which come from the motion of the test masses induced by the scalar gravitational wave), it was implicitly assumed that the propagation of the photon between the beam-splitter and the mirror of our interferometer is uniform as if it were moving in a flat space-time. But the presence of the tidal forces indicates that the space-time is curved. As a result another effect after the previous has to be considered, which requires spacial separation [12, 15].

For this effect we consider the interval for photons propagating along the z -axis

$$ds^2 = g_{00} dt^2 + dz^2. \quad (29)$$

The condition for a null trajectory ($ds = 0$) gives the coordinate velocity of the photons

$$v^2 \equiv \left(\frac{dz}{dt}\right)^2 = 1 + 2V(t, z), \quad (30)$$

which to first order in Φ is approximated by

$$v \approx \pm[1 + V(t, z)], \quad (31)$$

with $+$ and $-$ for the forward and return trip respectively. Knowing the coordinate velocity of the photon, the propagation time for its travelling between the beam-splitter and the mirror can be defined:

$$T_1(t) = \int_{z_b(t-T_1)}^{z_m(t)} \frac{dz}{v} \quad (32)$$

and

$$T_2(t) = \int_{z_m(t-T_2)}^{z_b(t)} \frac{(-dz)}{v}. \quad (33)$$

The calculations of these integrals would be complicated because the boundary $z_m(t)$ is changing with time. In fact it is

$$z_b(t) = \delta z_b(t) = 0 \quad (34)$$

but

$$z_m(t) = L + \delta z_m(t). \quad (35)$$

But, to first order in Φ , this contribution can be approximated by $\delta L_2(t)$ (see eq. (25)). Thus, the combined effect of the varying boundary is given by $\delta_1 T(t)$ in eq. (28). Then, only the times for photon propagation between the fixed boundaries 0 and L have to be calculated. Such propagation times will be denoted with $\Delta T_{1,2}$ to distinguish from $T_{1,2}$. In the forward trip, the propagation time between the fixed limits is

$$\Delta T_1(t) = \int_0^L \frac{dz}{v(t', z)} \approx T - \int_0^L V(t', z) dz, \quad (36)$$

where t' is the retardation time which corresponds to the unperturbed photon trajectory:

$$t' = t - (L - z)$$

(i.e. t is the time at which the photon arrives in the position L , so $L - z = t - t'$).

Similiary, the propagation time in the return trip is

$$\Delta T_2(t) = T - \int_L^0 V(t', z) dz, \quad (37)$$

where now the retardation time is given by

$$t' = t - z.$$

The sum of $\Delta T_1(t - T)$ and $\Delta T_2(t)$ gives the round-trip time for photons traveling between the fixed boundaries. Then the deviation of this round-trip time (distance) from its unperturbed value $2T$ is

$$\delta_2 T(t) = \int_0^L [V(t - 2T + z, z) + V(t - z, z)] dz. \quad (38)$$

From eqs. (14) and (38) it results:

$$\begin{aligned}
\delta_2 T(t) &= \frac{1}{2} m^2 \int_0^L [\int_0^z \Phi(t - 2T + a - v_G a) da + \int_0^z \Phi(t - a - v_G a) da] dz = \\
&= \frac{1}{4} m^2 \int_0^L [\Phi(t - v_G z - 2T + z) + \Phi(t - v_G z - z)] z^2 dz + \\
&\quad - \frac{1}{4} m^2 \int_0^L [\int_0^z \Phi'(t - 2T + a - v_G a) z^2 da + \int_0^z \Phi'(t - a - v_G a) z^2 da] dz,
\end{aligned} \tag{39}$$

Thus the total round-trip proper distance in presence of the scalar gravitational wave is:

$$T = 2T + \delta_1 T + \delta_2 T. \tag{40}$$

Now, to obtain the interferometer response function of the massive scalar field, the analysis can be transled in the frequency domine.

Using the Fourier transform of ψ defined from

$$\tilde{\psi}(\omega) = \int_{-\infty}^{\infty} dt \psi(t) \exp(i\omega t), \tag{41}$$

and recalling a theorem about Fourier transforms [15], it is simple to obtain:

$$\tilde{\psi}(\omega) = -\frac{\tilde{\Phi}(\omega)}{\omega^2}, \tag{42}$$

where

$$\tilde{\Phi}(\omega) = \int_{-\infty}^{\infty} dt \Phi(t) \exp(i\omega t). \tag{43}$$

is the Fourier transform of our scalar field. Then, in the frequency space, it results [15]:

$$\tilde{T}(\omega) = \Upsilon_l(\omega) \tilde{\Phi}(\omega), \tag{44}$$

where

$$\begin{aligned}
\Upsilon_l(\omega) &\equiv (1 - v_G^2) \exp[i\omega L(1 + v_G)] + \frac{1}{2\omega L(v_G^2 - 1)^2} \\
&\quad [\exp[2i\omega L](v_G + 1)^3(-2i + \omega L(v_G - 1) + 2 \exp[i\omega L(1 + v_G)]) \\
&\quad (6iv_G + 2iv_G^3 - \omega L + \omega L v_G^4) + (v_G + 1)^3(-2i + \omega L(v_G + 1))],
\end{aligned} \tag{45}$$

is the response function of an arm of our interferometer located in the z -axis, due to the longitudinal component of the massive scalar gravitational wave propagating in the same direction of the axis.

For $v_G \rightarrow 1$ it is $\Upsilon_l(\omega) \rightarrow 0$.

5 Conclusions

In this paper, which is a review of a previous research on gravitational waves from the R^{-1} high order theory of gravity [15], it has been shown that massive scalar modes of gravitational waves from the R^{-1} theory generate a longitudinal force in addition of the transverse one which is proper of the massless gravitational waves and the response of an arm of an interferometer to this longitudinal effect in the frame of a local observer has been computed. Important consequences from a theoretical point of view could arise from this approach, because it opens to the possibility of using the signals seen from interferometers to understand which is the correct theory of gravitation.

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